Name Jenny

Student Number

STA 442/2101 F 2014 Quiz 11

1. (6 points) A definition of the non-central chi-squared distribution is that if $Z \sim N(\mu, 1)$, then $Z^2 \sim \chi^2_{nc}(\nu = 1, \lambda = \mu^2)$.

   Suppose $Y_1, \ldots, Y_n$ are independent Binomial$(1, \theta)$. Then $Z_n^2 = \frac{n(\overline{Y}-\theta_0)^2}{\overline{Y}(1-\overline{Y})}$ is a statistic for testing $H_0 : \theta = \theta_0$ against the alternative that $\theta \neq \theta_0$. Under the alternative, Z_n^2 has an approximate non-central chi-squared distribution. Give the non-centrality parameter and justify it in terms of the definition above. Show your work. It is okay to write $Z_n^2 \approx \frac{n(\overline{Y}-\theta_0)^2}{\theta(1-\theta)}$.

$\bar{Y} \overset{approx}{\sim} N\left(\theta, \frac{\theta(1-\theta)}{n}\right)$, so

$\bar{Y} - \theta_0 \overset{approx}{\sim} N\left(\theta - \theta_0, \frac{\theta(1-\theta)}{n}\right)$, so

$$Z_n = \frac{\sqrt{n}(\bar{Y}-\theta_0)}{\sqrt{\bar{Y}(1-\bar{Y})}} \approx \frac{\sqrt{n}(\bar{Y}-\theta_0)}{\sqrt{\theta(1-\theta)}}$$

$\sim N\left(\frac{\sqrt{n}(\theta-\theta_0)}{\sqrt{\theta(1-\theta)}}, 1\right)$, and

$$Z^2 \sim \chi^2\left(1, \mu^2 = \frac{n(\theta-\theta_0)^2}{\theta(1-\theta)}\right.$$

2. (4 points) In Question 3g of the homework, you were asked for the power to detect an effect of bacteria type at cool temperatures. The answer is a number. Write the number in the space below. On your printout, circle the number and write "Question 2" beside it.

$\boxed{0.9927}$

Please attach your R printout.