Name ___Jenny___

Student Number _____

STA 431s13 Quiz 8

1. (3 points) In the following model, all expected values are zero and all error terms are independent. The variables Y_1, Y_2 and Y_3 are observable.
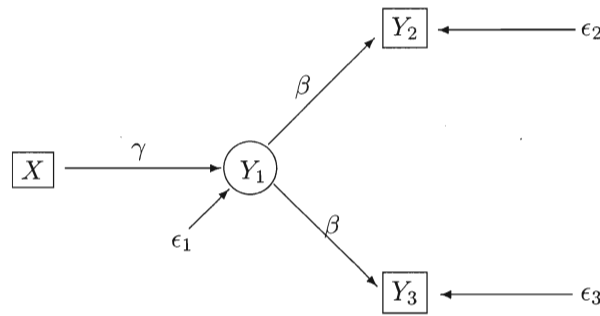
$$\begin{aligned} Y_2 &= \beta_1 Y_1 + \epsilon_2 \\ Y_3 &= \beta_2 Y_2 + \epsilon_3 \\ Y_1 &= \beta_3 Y_3 + \epsilon_1 \end{aligned}$$

Make a path diagram. If there is no coefficient on an arrow, it means the coefficient equals one. Give only one path diagram. If you make a false start, scratch it out or erase it completely.
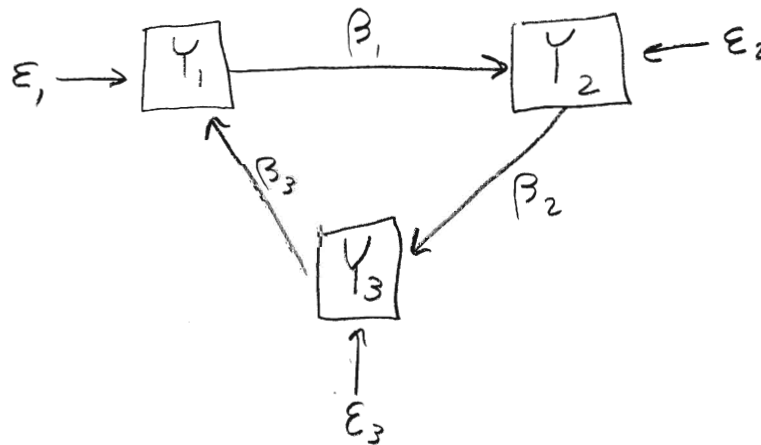
2. In the following model, all expected values are zero and there are no intercepts.

(a) (3 points) Write the model equations.

$$Y_1 = \gamma X + \varepsilon_1$$
$$Y_2 = \beta Y_1 + \varepsilon_2$$
$$Y_3 = \beta Y_1 + \varepsilon_3$$

(b) (1 point) List the exogenous variables (including error terms) and give their variances. Exogenous variables only, please! This is just to clarify your notation for the next part.

$$V(X) = \phi \;,\quad V(\varepsilon_1) = \psi_1 \;,\quad V(\varepsilon_2) = \psi_2 \;,\quad V(\varepsilon_3) = \psi_3$$

(c) (2 points) Calculate the covariance matrix of the observable variables in terms of the model parameters. Include *only* the observable variables. Even if everything else is correct, you will lose one mark for each latent variable you include in the covariance matrix.

$$Y_2 = \beta \gamma X + \beta \varepsilon_1 + \varepsilon_2$$
$$Y_3 = \beta \gamma X + \beta \varepsilon_1 + \varepsilon_3$$

	X	Y_2	Y_3
X	ϕ	$\beta \gamma \phi$	$\beta \gamma \phi$
Y_2		$\beta^2 \gamma^2 \phi + \beta^2 \psi_1 + \psi_2$	$\beta^2 \gamma^2 \phi + \beta^2 \psi_1$
Y_3			$\beta^2 \gamma^2 \phi + \beta^2 \psi_1 + \psi_3$

(d) (1 point) This model imposes an equality constraint on the covariances. What is it? Your answer is an equality involving σ_{ij} quantities.

$$\sigma_{12} = \sigma_{13}$$