Name Jerry

Student Number

STA 431s13 Quiz 7

1. ~~(7 points)~~ In this version of double measurement regression, the $q \times 1$ response vector $\mathbf{Y}_i$ is observable, while the $p \times 1$ explanatory vector $\mathbf{X}_i$ is latent and measured with error. The $p \times 1$ vectors $\mathbf{W}_{i,1}$ and $\mathbf{W}_{i,2}$ are observable. Independently for $i = 1, \ldots, n$, let

$$\begin{aligned} \mathbf{W}_{i,1} &= \mathbf{X}_i + \mathbf{e}_{i,1} \\ \mathbf{W}_{i,2} &= \mathbf{X}_i + \mathbf{e}_{i,2}, \\ \mathbf{Y}_i &= \boldsymbol{\beta}\mathbf{X}_i + \boldsymbol{\epsilon}_i, \end{aligned}$$

where all expected values are zero, $V(\mathbf{X}_i) = \boldsymbol{\Phi}$, $V(\mathbf{e}_{i,1}) = \boldsymbol{\Omega}_1$, $V(\mathbf{e}_{i,2}) = \boldsymbol{\Omega}_2$ and $V(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$. The random vectors $\mathbf{X}_i$, $\mathbf{e}_{i,1}$, $\mathbf{e}_{i,2}$ and $\boldsymbol{\epsilon}_i$ are all independent of one another.

1 pt

(a) Give the dimensions of the matrices below:

	Number of rows	Number of columns
$\boldsymbol{\Phi}$	p	p
$\boldsymbol{\Omega}_1$	p	p
$\boldsymbol{\Omega}_2$	p	p
β ~~$\boldsymbol{\Psi}$~~	q	p
ψ ~~$\boldsymbol{\beta}$~~	~~p~~ q	q

1 pt

(b) Calculate $C(\mathbf{W}_{i,1}, \mathbf{Y}_i)$. Show your work. **Circle your final answer.**

$$C(W_1, Y) = E(W_1 Y') = E(X+e_1)(\beta X + \varepsilon)'$$

$$= E(X+e_1)(X'\beta' + \varepsilon') = E(XX'\beta') + E(X\varepsilon') + E(eX'\beta') + E(e_1\varepsilon')$$

$$= E(XX')\beta' + E(X)E(\varepsilon') + E(e)E(X')\beta' + E(e_1)E(\varepsilon')$$

$$= \Phi\beta' + 0 + 0 + 0$$

$$= \Phi\beta'$$

$$\begin{aligned}
\mathbf{W}_{i,1} &= \mathbf{X}_i + \mathbf{e}_{i,1} \\
\mathbf{W}_{i,2} &= \mathbf{X}_i + \mathbf{e}_{i,2}, \\
\mathbf{Y}_i &= \beta \mathbf{X}_i + \epsilon_i,
\end{aligned}$$

1 pt

(c) Fill in the main diagonal and the upper triangular part of the partitioned covariance matrix below. You don't have to show any work.

	$\mathbf{W}_1$	$\mathbf{W}_2$	$\mathbf{Y}$
$\mathbf{W}_1$	$\Phi + \Omega_1$	Φ	$\Phi \beta'$
$\mathbf{W}_2$		$\Phi + \Omega_2$	$\Phi \beta'$
$\mathbf{Y}$			$\beta \Phi \beta' + \Psi$

$$= \left(\begin{array}{c|c|c} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \hline & \Sigma_{22} & \Sigma_{23} \\ \hline & & \Sigma_{33} \end{array} \right)$$

2 pts

(d) Prove that all the parameter matrices are identifiable. Show your work. For reference, the matrices are listed in Question 1a.

$$\Phi = \Sigma_{12}$$

$$(\Phi^{-1} \Sigma_{13})' = \Sigma_{13}' \Phi^{-1} = \beta \Phi \Phi^{-1} = \beta,$$

So $$\beta = \Sigma_{13}' \Phi^{-1}$$

$$\Omega_1 = \Sigma_{11} - \Phi \qquad \Omega_2 = \Sigma_{22} - \Phi$$

$$\Psi = \Sigma_{33} - \beta \Phi \beta'$$

✓

(e) (1 point) How many equality constraints are imposed on $\boldsymbol{\Sigma}$ by the model? The answer is an expression in p and q. **Circle your answer.**

$\Sigma_{13} = \Sigma_{23}$ & Σ_{12} symmetric

$pq + \frac{1}{2} p(p-1)$

2. Please refer to your printout for the pig data.

(a) (2 points) For farm i, let $e_{i,1}$ refer to the measurement error in the number of breeding sows present according to questionnaire one, and let $e_{i,2}$ refer to the measurement error in the number of sows giving birth according to questionnaire one. Similarly, $e_{i,3}$ is the measurement error in the number of breeding sows present according to questionnaire two, and $e_{i,4}$ is the measurement error in the number of sows giving birth according to questionnaire two. Denote the measurement error covariance matrices by $V((e_{i,1}, e_{i,2})') = \boldsymbol{\Omega}_1$ and $V((e_{i,3}, e_{i,4})') = \boldsymbol{\Omega}_2$ (this is different from the meaning of these matrices in Question 1). You conducted a Wald test to see whether the two covariance matrices are equal or not. Give the Wald chi-squared statistic, the degrees of freedom and the p-value. These are numbers from your printout.

$\chi^2 = 41.7$, $df = 3$, $p < 0.0001$

(b) (2 points) Denote the variance of the true number of breeding sows by ϕ. Give a numerical estimate of ϕ that is *not the MLE*. The answer is a number that will be on your printout if you used the `pcorr` option. Write the number in the space below.

348.53

Please attach your log file and your list file to the quiz paper. Make sure your name is written on both printouts.