Name Jerry

Student Number

STA 431s13 Quiz 6

1. (3 points) In the following model, $X_{i,1}$, $X_{i,2}$ and Y_i are latent variables, while $W_{i,1}$, $W_{i,1}$ and V_i are observable. Independently for $i = 1, \ldots, n$, let

$$\begin{aligned} Y_i &= \beta X_{i,1} + \epsilon_i \\ W_{i,1} &= X_{i,1} + e_{i,1} \\ W_{i,2} &= X_{i,2} + e_{i,2} \\ V_i &= Y_i + e_{i,3} \end{aligned}$$

where $\epsilon_i \sim N(0, \psi)$, $e_{i,j} \sim N(0, \omega_j)$ for $j = 1, 2, 3$, and the vector $\mathbf{X}_i = (X_{i,1}, X_{i,2})'$ is bivariate normal with expected value zero and covariance matrix

$$V \left(\begin{array}{c} X_{i,1} \\ X_{i,2} \end{array} \right) = \left(\begin{array}{cc} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{array} \right).$$

The error term are independent of one another, and independent of $\mathbf{X}_i$.

(a) What is the parameter vector $\boldsymbol{\theta}$ for this model?

$\theta = (\beta, \psi, \omega_1\ \omega_2\ \omega_3\ \phi_{11}\ \phi_{12}\ \phi_{22})$

(b) Does this problem pass the test of the Parameter Count Rule? Answer Yes or No and give the numbers. No. There are 8 parameters and 6 ~~covarian~~ variance-covariances

(c) Write down the covariance matrix of the observable variables. *You do not need to show any work.*

	W_1	W_2	V
W_1	$\phi_{11} + \omega_1$	ϕ_{12}	$\beta\phi_{11}$
W_2		$\phi_{22} + \omega_2$	$\beta\phi_{12}$
V			$\beta^2\phi_{11} + \psi + \omega_3$

(d) In this data set, X_2 is a tool for identifying β, and is carefully chosen to be related to X_1. Denoting the covariance matrix of the observable data by $\mathbf{\Sigma} = [\sigma_{ij}]$, what hypothesis could you test about the σ_{ij} to verify this?

$H_0: \sigma_{12} = 0$

(e) Assuming $\phi_{12} \neq 0$, show that β is identifiable.

$$\frac{\sigma_{23}}{\sigma_{12}} = \frac{\beta\phi_{12}}{\phi_{12}} = \beta$$

(f) Still Assuming $\phi_{12} \neq 0$, give a reasonable estimator of β. Warning: An estimator is a *statistic*, a function of the sample data. If you write your estimator as a function of any unknown parameters, you get no marks for this part.

$$\hat{\beta} = \frac{\hat{\sigma}_{23}}{\hat{\sigma}_{12}}$$

(g) How do you know that your estimator is consistent? You may use the (strong) consistency of sample variances and covariances without proof.

$\hat{\sigma}_{23} \xrightarrow{as} \sigma_{23}$, $\hat{\sigma}_{12} \xrightarrow{as} \sigma_{12} \neq 0$, so

$\hat{\beta} = \frac{\hat{\sigma}_{23}}{\hat{\sigma}_{12}} \xrightarrow{as} \frac{\sigma_{23}}{\sigma_{12}} = \beta$ by continuity

2. (3 points) In your analysis of the longitudinal IQ data, you tested the null hypothesis of equality for the four regression coefficients connecting birth mother's IQ to her child's IQ at 4 ages.

(a) Show the calculation of G^2 below, including the two numbers you subtracted to get it. **circle your G^2 value.**

	χ^2	Objective functions
Version 9.1	25.6474 − 10.0006 = 15.65	62 * (0.4204488137 − 0.1639435398) 15.903
Version 9.3	25.6474 − 10.0006 = 15.65	62 * (0.4204488137 − 0.1639435398)

(b) With a critical chi-squared value of 7.815, what do you conclude from the test? Your answer is a statement about IQ scores.

The connection between birth mother's IQ & child's IQ depends on ~~age~~ child's age.

Please attach your log file and your list file to the quiz paper. Make sure your name is written on both printouts.