Student Number _

STA 431s13 Quiz 6

1. (3 points) In the following model, $X_{i,1}$, $X_{i,2}$ and Y_i are latent variables, while $W_{i,1}$, $W_{i,1}$ and V_i are observable. Independently for $i = 1, \ldots, n$, let

$$Y_i = \beta X_{i,1} + \epsilon_i$$

 $W_{i,1} = X_{i,1} + e_{i,1}$
 $W_{i,2} = X_{i,2} + e_{i,2}$
 $V_i = Y_i + e_{i,3}$

where $\epsilon_i \sim N(0, \psi)$, $e_{i,j} \sim N(0, \omega_j)$ for j = 1, 2, 3, and the vector $\mathbf{X}_i = (X_{i,1}, X_{i,2})'$ is bivariate normal with expected value zero and covariance matrix

$$V\left(\begin{array}{c} X_{i,1} \\ X_{i,2} \end{array}\right) = \left(\begin{array}{cc} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{array}\right).$$

The error term are independent of one another, and independent of X_i .

(a) What is the parameter vector $\boldsymbol{\theta}$ for this model?

- (b) Does this problem pass the test of the Parameter Count Rule? Answer Yes or No and give the numbers. No. There are 8 parameters and 6
- (c) Write down the covariance matrix of the observable variables. You do not need to show any work.

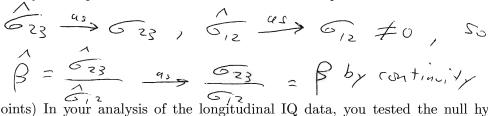
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(d) In this data set, X_2 is a tool for identifying β , and is carefully chosen to be related to X_1 . Denoting the covariance matrix of the observable data by $\Sigma = [\sigma_{ij}]$, what hypothesis could you test about the σ_{ij} to verify this?

(e) Assuming $\phi_{12} \neq 0$, show that β is identifiable.

$$\frac{G_{23}}{G_{12}} = \frac{\beta \varphi_{12}}{\varphi_{12}} = \beta$$

- (f) Still Assuming $\phi_{12} \neq 0$, give a reasonable estimator of β . Warning: An estimator is a statistic, a function of the sample data. If you write your estimator as a function of any unknown parameters, you get no marks for this part. B = 523
- (g) How do you know that your estimator is consistent? You may use the (strong) consistency of sample variances and covariances without proof.



- 2. (3 points) In your analysis of the longitudinal IQ data, you tested the null hypothesis of equality for the four regression coefficients connecting birth mother's IQ to her child's IQ at 4 ages.
 - (a) Show the calculation of G^2 below, including the two numbers you subtracted to get it.

(a) Show the calculation of
$$G^2$$
 below, including the two numbers you subtracted to get it. circle your G^2 value.

Objective functions

Varion 9.1

Varion 9.3

 $25.6474 - 10.0006$
 $25.6474 - 10.0006$
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 $25.6474 - 10.0006$

(b) With a critical chi squared value of 7.815 , what do you canclude from the test? Your

(b) With a critical chi-squared value of 7.815, what do you conclude from the test? Your answer is a statement about IQ scores.

Please attach your log file and your list file to the quiz paper. Make sure your name is written on both printouts.