Name Jerry

Student Number

STA 431s13 Quiz 4

1. Independently for $i = 1, \ldots, n$, let

$$\begin{aligned} Y_i &= \beta X_i + \epsilon_i \\ W_i &= X_i + e_i, \end{aligned}$$

where $E(X_i) = \mu_x \neq 0$, $E(\epsilon_i) = E(e_i) = 0$, $Var(X_i) = \phi$, $Var(\epsilon_i) = \psi$, $Var(e_i) = \omega$, and X_i, e_i andϵ_i are all independent. The variables X_i is latent, while W_i and Y_i are observable.

(a) (1 point) Assuming normality, what is the parameter vector $\boldsymbol{\theta}$ for this model?

$\underset{\sim}{\theta} = (\beta, \mu_x, \phi, \psi, \omega)$

(b) (1 point) Does this model pass the test of the parameter count rule? Answer Yes or No and give the numbers.

Yes, there are 5 parameters and 2+3 moments

(c) (5 points) Let

$$\widehat{\beta}_n = \frac{\sum_{i=1}^n Y_i}{\sum_{i=1}^n W_i}.$$

Is $\widehat{\beta}_n$ a consistent estimator of β? Answer Yes or No and prove your answer.

Yes

By the Law of Large Numbers, $\overline{Y}_n \xrightarrow{as} E(Y_i) = \beta\mu_x + 0$ and $\overline{W}_n \xrightarrow{as} E(W_i) = \mu_x + 0$. Then

$$\widehat{\beta}_n = \frac{\frac{1}{n}\sum_{i=1}^n Y_i}{\frac{1}{n}\sum_{i=1}^n W_i} = \frac{\overline{Y}_n}{\overline{W}_n} \xrightarrow{as} \frac{\beta\mu_x}{\mu_x} = \beta$$

by continuous mapping

2. (3 points) For the SAT data, what is maximum likelihood estimate of the variance of the error term in your regression? The answer is a number from your printout. **Write the number below and also circle it on your printout**. Do not answer this question if you don't have a printout.

0.29767

Please attach your log file and your list file to the quiz paper. Make sure your name is written on both printouts.