

STA 431s13 Quiz 3

Recall $\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$, and if

$W = X + e$, the reliability of W is defined as $\text{Corr}(W, X)^2 = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2}$.

1. (6 points) Suppose we have two equivalent measurements with uncorrelated measurement error:

$$W_1 = \nu + X + e_1$$

$$W_2 = \nu + X + e_2,$$

where $E(X) = \mu$, $\text{Var}(X) = \sigma_X^2$, $E(e_1) = E(e_2) = 0$, $\text{Var}(e_1) = \text{Var}(e_2) = \sigma_e^2$, and X , e_1 and e_2 are all independent. Let $S = W_1 + W_2$.

- (a) Calculate the reliability of S . Show your work. You may use the centering rule if you wish.

$$S = 2\nu + 2X + e_1 + e_2, \quad \text{Var}(S) = 4\sigma_X^2 + 2\sigma_e^2$$

$$\text{Cov}(S, X) = \text{Cov}(2\hat{X}, \hat{X}) = E(\hat{S}\hat{X})$$

$$= E(2\hat{X} + e_1 + e_2)(\hat{X}) = 2E(\hat{X}^2) + E(e_1)E(\hat{X}) + E(e_2)E(\hat{X})$$

$$= 2\sigma_X^2, \text{ so}$$

$$\text{Reliability of } S \text{ is } \text{Corr}(S, X)^2 = \left[\frac{\text{Cov}(S, X)}{\sqrt{\text{Var}(S)} \sqrt{\text{Var}(X)}} \right]^2$$

$$= \left[\frac{2\sigma_X^2}{\sqrt{4\sigma_X^2 + 2\sigma_e^2} \sqrt{\sigma_X^2}} \right]^2$$

$$= \frac{4\cancel{\sigma_X^2} \cancel{\sigma_X^2}}{(4\sigma_X^2 + 2\sigma_e^2) \cancel{\sigma_X^2}}$$

$$= \frac{4\cancel{\sigma_X^2}}{4(\sigma_X^2 + \frac{1}{2}\sigma_e^2)}$$

$$= \frac{\sigma_X^2}{\sigma_X^2 + \frac{1}{2}\sigma_e^2}$$

- (b) Which is greater, the reliability of W_1 , or the reliability of S ? Answer the question and show some work. The reliability of W_1 is given on side one; you don't have to calculate it

Reliability of S is greater:

$$\frac{\sigma_x^2}{\sigma_x^2 + \sigma_e^2} < \frac{\sigma_x^2}{\sigma_x^2 + \frac{1}{2}\sigma_e^2}$$

This is enough.
Dividing by a smaller number! Or show work.

$$\Leftrightarrow \frac{1}{\sigma_x^2 + \sigma_e^2} < \frac{1}{\sigma_x^2 + \frac{1}{2}\sigma_e^2}$$

$$\Leftrightarrow \sigma_x^2 + \sigma_e^2 > \sigma_x^2 + \frac{1}{2}\sigma_e^2$$

$$\Leftrightarrow \sigma_e^2 > \frac{1}{2}\sigma_e^2$$

2. (4 points) For the SAT data, what is the sample standard deviation of Grade Point Average? The answer is a number from your printout. **Circle the number on your printout.** Do not answer this question if you don't have a printout.

0.5802910

Please attach your log file and your list file to the quiz paper. Make sure your name is written on both printouts.