Name Jerry

Student Number

STA 431s13 Quiz 2

$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$, or $E(g(X)) = \sum_x g(x)\, p_X(x)$

$Var(X) = E[(X - \mu_X)^2]$ $\quad$ $Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]$

$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$ $\quad$ $\mathbf{\Sigma} = \mathbf{P\Lambda P}'$

$V(\mathbf{X}) = E\left\{(\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{X} - \boldsymbol{\mu}_x)'\right\}$ $\quad$ $C(\mathbf{X},\mathbf{Y}) = E\left\{(\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{Y} - \boldsymbol{\mu}_y)'\right\}$

If $\mathbf{X} \sim N(\boldsymbol{\mu}, \mathbf{\Sigma})$, then $\mathbf{AX} \sim N(\mathbf{A}\boldsymbol{\mu}, \mathbf{A\Sigma A}')$.

1. (4 points) Show that the trace of a square symmetric matrix is the sum of its eigenvalues.

$$tr(\Sigma) = tr(\underbrace{P}_{A}\underbrace{\Lambda P'}_{B}) = tr(\Lambda P'P)$$

$$= tr(\Lambda) = \sum_j \lambda_j$$

2. (4 points) Let $\mathbf{X}$ be a random vector with expected value $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$, and let $\mathbf{A}$ be a matrix of constants. Derive an expression for $V(\mathbf{AX})$. Start with the definition of a variance-covariance matrix from the formula sheet.

$$E(AX) = A\mu$$

$$\begin{aligned} V(AX) &= E\{(AX - A\mu)(AX - A\mu)'\} \\ &= E\{A(X-\mu)[A(X-\mu)]'\} \\ &= E\{A(X-\mu)(X-\mu)'A'\} \\ &= A\,E\{(X-\mu)(X-\mu)'\}A' \\ &= A\Sigma A' \end{aligned}$$

3. (2 points) In homework, you were asked to obtain a numerical maximum likelihood for the parameter α of a one-parameter gamma distribution.

   (a) Write the number from your printout in the space below.

   (b) Attach your printout to the quiz. Make sure your name is written on the printout. **Circle the MLE $\widehat{\alpha}$ on your printout.**