

Name Jerry

Student Number _____

STA 431s13 Quiz 1

1. (4 points) Let \mathbf{X} be an n by p matrix with $n \neq p$, and suppose that $(\mathbf{X}'\mathbf{X})^{-1}$ exists. Why is it incorrect to say that $(\mathbf{X}'\mathbf{X})^{-1} = \mathbf{X}^{-1}\mathbf{X}'^{-1}$? You have a lot more room than you need for the answer.

\mathbf{X} is not a square matrix, so \mathbf{X}^{-1} is not even defined.

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x) dx, \quad \text{or } E(g(X)) = \sum_x g(x) p_X(x)$$

$$\text{Var}(X) = E[(X - \mu_X)^2] \quad \text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

2. (6 points) Let X and Y be random variables. Calculate $\text{Var}(X + Y)$ in terms of variances and covariances. You may use the linearity of expected value, but do *not* use the centering rule.

$$E(X + Y) = E(X) + E(Y) = \mu_X + \mu_Y$$

$$\text{Var}(X + Y) = E\left\{ (X + Y - (\mu_X + \mu_Y))^2 \right\}$$

$$= E\left\{ ((X - \mu_X) + (Y - \mu_Y))^2 \right\}$$

$$= E\left\{ (X - \mu_X)^2 + 2(X - \mu_X)(Y - \mu_Y) + (Y - \mu_Y)^2 \right\}$$

$$= E\{(X - \mu_X)^2\} + 2E\{(X - \mu_X)(Y - \mu_Y)\} + E\{(Y - \mu_Y)^2\}$$

$$= \text{Var}(X) + 2\text{Cov}(X, Y) + \text{Var}(Y)$$