tests of overidentifying restrictions. (Perhaps we should note that such a test will not always involve the t-statistic for a single coefficient but may lead to the F-test for a whole set of coefficients.) This rule does not cover all cases, as will become apparent when we consider models for which OLS is not the appropriate method of estimation.

It is vital to keep the matter of tests of overidentifying restrictions in perspective. Valuable as such tests may be, they do not really bear upon what may be the most problematical issue in the specification of a recursive model, that is, the causal ordering of the variables. It is the gravest kind of fallacy to suppose that, from a number of competing models involving different causal orderings, one can select the true model by finding the one that comes closest to satisfying a particular test of overidentifying restrictions. (Examples of such a gross misunderstanding of the Simon–Blalock technique can be found, among other places, in the political science literature of the mid-1960s.) In fact, a test of the causal ordering of variables is beyond the capacity of any statistical method; or, in the words of Sir Ronald Fisher (1946), "if . . . we choose a group of social phenomena with no antecedent knowledge of the causation or absence of causation among them, then the calculation of correlation coefficients, total or partial, will not advance us a step toward evaluating the importance of the causes at work [p. 191]."

Exercise. *Show how to express all the correlations in terms of path coefficients in the model on page 44, using Sewall Wright's multiplication rule for reading a path diagram.*

Exercise. *Change the model on page 28 so that p_{43} is specified to be zero. Draw the revised path diagram. Obtain an expression for the overidentifying restriction. Indicate how one would estimate the coefficients in the revised model, supposing the overidentifying restriction was not called seriously into question.*

FURTHER READING

See Walker and Lev (1953, Chapter 13) for a treatment of multiple regression with standardized variables. Sociological studies using recursive models are numerous; for examples, see Blalock (1971, Chapter 7) and Duncan, Featherman, and Duncan (1972).

4

Structural Coefficients in Recursive Models

In Chapter 3 a four-variable recursive model was formulated in terms of standardized variables. That procedure has some advantages.

(*1*) Certain algebraic steps are simplified.
(*2*) Sewall Wright's rule for expressing correlations in terms of path coefficients can be applied without modification.
(*3*) Continuity is maintained with the earlier literature on path analysis and causal models in sociology.
(*4*) It shows how an investigator whose data are available only in the form of a correlation matrix can, nevertheless, make use of a clearly specified model in interpreting those correlations.

Despite these advantages (see also Wright, 1960), it would probably be salutary if research workers relinquished the habit of expressing variables in standard form. The main reason for this recommendation is that standardization tends to obscure the distinction between the structural coefficients of the model and the several variances and covariances that describe the joint distribution of the variables in a certain population.

Although it will involve some repetition of ideas, we will present the four-variable recursive model again, this time avoiding the stipulation that all variables have unit variance. The model is

$$(x_1 \text{ exogenous})$$

$$x_2 = b_{21}x_1 + u$$

$$x_3 = b_{32}x_2 + b_{31}x_1 + v$$

$$x_4 = b_{43}x_3 + b_{42}x_2 + b_{41}x_1 + w$$

We continue to assume that all variables have zero expectation; unlike standardization, this achieves a useful simplification without significant loss of generality or confusion of issues. (Only in special cases, where "regression through the origin" is involved, does this stipulation require modification.)

The specification on the disturbances is that the disturbance in each equation has zero covariance with the predetermined variables in that equation and all "earlier" equations. The one strictly exogenous variable x_1 is predetermined in each equation. In addition, x_2 is a predetermined variable in the x_3-equation and the x_4-equation, while x_3 is a predetermined variable in the x_4-equation. Thus we specify $E(x_1 u) = E(x_1 v) = E(x_1 w) = E(x_2 v) = E(x_2 w) = E(x_3 w) = 0$. We find, as a consequence of this specification, that it also is the case that $E(uv) = E(uw) = E(vw) = 0$.

(We note, for future reference, that in both recursive and nonrecursive models, the usual specification is zero covariance of predetermined variables in an equation with the disturbance of that equation. In the case of recursive models this generally implies zero covariances among the disturbances of different equations. This does not, however, hold true for nonrecursive models.)

To deduce properties of the model from the specifications regarding its functional form and its disturbances, we multiply through equations of the model by variables in the model, and take expectations. For convenience, we denote the variance $E(x_j^2)$ by σ_{jj}, using the double subscript in place of the exponent of the usual notation, σ_j^2. Similarly, a covariance is denoted by $\sigma_{hj} = E(x_h x_j)$, $h \neq j$.

The normal equations are obtained by multiplying through each equation by its predetermined variables:

$$\sigma_{12} = b_{21}\sigma_{11} \qquad \text{(from the } x_2\text{-equation)}$$

$$\left.\begin{aligned} \sigma_{13} &= b_{31}\sigma_{11} + b_{32}\sigma_{12} \\ \sigma_{23} &= b_{31}\sigma_{12} + b_{32}\sigma_{22} \end{aligned}\right\} \qquad \text{(from the } x_3\text{-equation)}$$

$$\left.\begin{aligned} \sigma_{14} &= b_{41}\sigma_{11} + b_{42}\sigma_{12} + b_{43}\sigma_{13} \\ \sigma_{24} &= b_{41}\sigma_{12} + b_{42}\sigma_{22} + b_{43}\sigma_{23} \\ \sigma_{34} &= b_{41}\sigma_{13} + b_{42}\sigma_{23} + b_{43}\sigma_{33} \end{aligned}\right\} \qquad \text{(from the } x_4\text{-equation)}$$

Clearly, it is possible to solve uniquely for the b's, which we shall term the *structural coefficients*, in terms of the population variances and covariances. In practice, of course, the latter are unknown. Hence, we can only estimate the structural coefficients. If, in the normal equations, the σ's are replaced by sample moments, the estimates obtained are equivalent to those of ordinary least-squares (OLS) regression of x_2 on x_1; x_3 on x_2 and x_1; and x_4 on x_3, x_2, and x_1. By sample moments we mean the quantities

$$m_{jj} = \sum x_j^2$$

$$m_{hj} = \sum x_h x_j \qquad (h \neq j)$$

where the summation is over all the observations in the sample and each observation on x_j is expressed as a deviation from the mean of x_j in the sample.

Along with the preceding normal equations, we will find it useful to obtain expressions for the variances of the dependent variables by multiplying through each equation of the model by its dependent variable:

$$\sigma_{22} = b_{21}\sigma_{12} + \sigma_{2u}$$

$$\sigma_{33} = b_{32}\sigma_{23} + b_{31}\sigma_{13} + \sigma_{3v}$$

$$\sigma_{44} = b_{43}\sigma_{34} + b_{42}\sigma_{24} + b_{41}\sigma_{14} + \sigma_{4w}$$

These may be simplified slightly by noting that $\sigma_{2u} = \sigma_{uu}$, $\sigma_{3v} = \sigma_{vv}$, and $\sigma_{4w} = \sigma_{ww}$, which facts are deduced by multiplying through each equation of the model by its disturbance.

It is instructive to rewrite the expressions for the variances and covariances in the form given below (the algebra involves only straightforward, though tedious, substitutions in the equations already given). The variances may be written:

$$\sigma_{11} \text{ exogenous}$$

$$\sigma_{22} = b_{21}^2\sigma_{11} + \sigma_{uu}$$

$$\sigma_{33} = A_6\sigma_{11} + b_{32}^2\sigma_{uu} + \sigma_{vv}$$

$$\sigma_{44} = A_9\sigma_{11} + A_0\sigma_{uu} + b_{43}^2\sigma_{vv} + \sigma_{ww}$$

and the covariances:

$$\sigma_{12} = b_{21}\sigma_{11}$$

$$\sigma_{13} = A_1\sigma_{11}$$

$$\sigma_{23} = A_3\sigma_{11} + b_{32}\sigma_{uu}$$

$$\sigma_{14} = A_2\sigma_{11}$$

$$\sigma_{24} = A_4\sigma_{11} + A_5\sigma_{uu}$$

$$\sigma_{34} = A_7\sigma_{11} + A_8\sigma_{uu} + b_{43}\sigma_{vv}$$

where

$$A_1 = b_{31} + b_{32}b_{21}$$

$$A_2 = b_{41} + b_{42}b_{21} + b_{43}A_1$$

$$A_3 = b_{21}(b_{31} + b_{32}b_{21})$$

$$A_4 = b_{41}b_{21} + b_{42}b_{21}^2 + b_{43}A_3$$

$$A_5 = b_{42} + b_{43}b_{32}$$

$$A_6 = b_{32}A_3 + b_{31}A_1$$

$$A_7 = b_{41}A_1 + b_{42}A_3 + b_{43}A_6$$

$$A_8 = b_{32}(b_{42} + b_{43}b_{32})$$

$$A_9 = b_{43}A_7 + b_{42}A_4 + b_{41}A_2$$

$$A_0 = b_{42}A_5 + b_{43}A_8$$

The A's have been introduced merely to abbreviate the presentation and have no particular interpretation in themselves. It is important to note, however, that all the A's are nonlinear combinations of the structural coefficients (the b's) and *involve no other terms*. Thus, we can draw an important conclusion. The variances and covariances are all functions of (at most) three kinds of quantities: (*1*) the variance of the exogenous variable; (*2*) the variance(s) of one or more disturbances; and (*3*) a nonlinear combination of structural coefficients. Table 4.1

Table 4.1 Sources of Observable Variances and Covariances

Variance or covariance	*Is a function of* σ_{11}	σ_{uu}	σ_{vv}	σ_{ww}	b_{21}	b_{31}	b_{32}	b_{41}	b_{42}	b_{43}
σ_{11}	×	...	...	...	...	...	...	...	...	...
σ_{22}	×	×	...	...	×	...	...	...	...	...
σ_{33}	×	×	×	...	×	×	×	...	...	...
σ_{44}	×	×	×	×	×	×	×	×	×	×
σ_{12}	×	...	...	...	×	...	...	...	...	...
σ_{13}	×	...	...	...	×	×	×	...	...	...
σ_{14}	×	...	...	...	×	×	×	×	×	×
σ_{23}	×	×	...	...	×	×	×	...	...	...
σ_{24}	×	×	...	...	×	×	×	×	×	×
σ_{34}	×	×	×	...	×	×	×	×	×	×

makes this explicit in each instance. The first component (σ_{11}) is involved in all the variances and covariances. One or more of the disturbance variances (σ_{uu}, σ_{vv}, σ_{ww}) are involved in the variances of all the dependent variables in the model and in the covariances of these variables with each other. Some combination of structural coefficients (the b's) is involved in all variances (except σ_{11}) and covariances. Thus, it is possible to regard the variances and covariances of the observed variables as having arisen entirely from these three sources. Moreover, we can describe these components as separable, in the following sense. We can suppose without contradiction (that is, without violating any other property of the model) that one of these components may change without either of the others having to change. If any of them changes, however, the observable variances and covariances will, in general, change.

This remarkable property of the model should be considered carefully by the investigator, for it has some far-reaching implications.

Suppose we had two populations under study and we specified our three-equation model as holding in each. It could happen that the structural coefficients are the same in the two populations and the variances of the several disturbances are likewise the same. But if only σ_{11} differs between the two populations, we will observe differences in all the other variances and all the covariances. Incidentally, we will also observe differences (in general) in all the correlations in the two populations and also (in general) in all the standardized path coefficients, not to mention other purported measures of "relative importance" or "unique contribution" of variables. Thus the observable facts about the two populations (as reflected in sample estimates of variances, covariances, and correlations) will suggest that they differ in many ways. But the premise of the illustration is that they differ in only one way: with respect to the variance of the exogenous variable. The model is invariant across populations with respect to the structural coefficients and the variances of the disturbances.

Another possibility is that both σ_{11} and the disturbance variances differ as between two populations, but the structural coefficients are the same. Again we would observe entirely different variance–covariance (or correlation) matrices in the two populations, even though only four of the ten quantities in the column headings of the table actually differ.

The possibilities just described are only hypothetical. But there would not be much purpose in devising a model to use in interpreting data if we did not have some hope that at least some features of our model would be invariant with respect to some changes in the circumstances under which it is applied. If all the model is good for is to describe a particular set of data—if with any new set of data we will be obliged to change all the quantities listed in the column headings of the table, even though we continue to specify the same mathematical form of the model—then we might as well forego the effort of devising and estimating the model. It offers no economy of description, since there are as many parameters across the top of the table (neglecting the possibility that some b's may be zero and therefore may be omitted) as there are variances and covariances listed in the stub. We can transform the variances and covariances into the parameters, or vice versa, by mathematical operations already described in these pages. Hence, from a purely descriptive standpoint, we might as well let stand the first set of estimates we compute—the variances and covariances, or correlations—and not bother with the structural coefficients.

Another line of reflection is suggested by this analysis. It could happen that a macrosociological model proposed for a given society gave rise to (nearly) invariant estimates of structural coefficients over a period of time (for example, for several successive birth cohorts), even though the variances of the exogenous variable and the disturbances were changing. One would then observe "social change" with respect to variances and covariances of the dependent variables. But in another sense, no "social change" would be occurring, since the structural coefficients were staying constant. If, on the other hand, the latter should change, one would really be dealing with social change, in a deeper sense of the term. The first kind of social change would, in a sense, be "explained" by the model (though, of course, the model does not speak to the sources of change in the exogenous variable and disturbances themselves). The second kind of social change—modification of structural coefficients (or "structural change," if one likes)—cannot be explained in any sense by the model. Even so, one might argue, the model—if one held to it with good reason, despite changes in structural coefficients—would at least make clear what it is about the social changes occurring that requires explanation. But surely our scientific aspirations and efforts should be directed toward the construction of models which are themselves "explanatory" in a proper scientific sense of the word, and not merely in the sense of providing some parametrization of the descriptive statistics which serves merely as a clue to the task of scientific explanation.

We gain still another perspective on the concept of structural coefficients in learning how to transform the model into a different set of equations. We continue with the model presented at the beginning of this chapter. By straightforward substitution we eliminate x_2 from the x_3-equation and both x_2 and x_3 from the x_4-equation; the x_2-equation is repeated as it stands. These manipulations yield the following three equations as the *reduced form* of the model:

$$(x_1 \text{ exogenous})$$
$$x_2 = b_{21}x_1 + u$$
$$x_3 = (b_{31} + b_{32}b_{21})x_1 + b_{32}u + v$$
$$x_4 = [b_{41} + b_{42}b_{21} + b_{43}(b_{31} + b_{32}b_{21})]x_1 + (b_{42} + b_{43}b_{32})u + b_{43}v + w$$

To obtain a compact notation for the coefficients and disturbances of the reduced form, we rewrite the foregoing equations, making use of the following definitions:

$$a_{21} = b_{21}$$
$$a_{31} = b_{31} + b_{32}b_{21}$$
$$a_{41} = b_{41} + b_{42}b_{21} + b_{43}(b_{31} + b_{32}b_{21})$$
$$u' = u$$
$$v' = b_{32}u + v$$
$$w' = (b_{42} + b_{43}b_{32})u + b_{43}v + w$$

This yields

$$x_2 = a_{21}x_1 + u'$$
$$x_3 = a_{31}x_1 + v'$$
$$x_4 = a_{41}x_1 + w'$$

We find that the exogenous variable is uncorrelated with the reduced-form disturbances, since

$$E(x_1u') = E(x_1u) = 0$$
$$E(x_1v') = b_{32}E(x_1u) + E(x_1v) = 0$$
$$E(x_1w') = (b_{42} + b_{43}b_{32})E(x_1u) + b_{43}E(x_1v) + E(x_1w) = 0$$

making use of the initial specification on the disturbances in the model. However, in the reduced form (unlike the *structural form* in which the model was originally specified), it is no longer true that disturbances are uncorrelated among themselves. In fact, we can derive explicit expressions for the covariances among reduced-form disturbances (recalling that the covariances among structural-form disturbances are zero):

$$\sigma_{u'v'} = E(u'v') = b_{32}E(u^2) + E(uv) = b_{32}\sigma_{uu}$$
$$\sigma_{u'w'} = (b_{42} + b_{43}b_{32})\sigma_{uu}$$
$$\sigma_{v'w'} = b_{32}(b_{42} + b_{43}b_{32})\sigma_{uu} + b_{43}\sigma_{vv}$$

The reduced-form disturbance variances likewise are functions of structural coefficients and variances of structural-form disturbances:

$$\sigma_{u'u'} = \sigma_{uu}$$
$$\sigma_{v'v'} = b_{32}^2\sigma_{uu} + \sigma_{vv}$$
$$\sigma_{w'w'} = (b_{42} + b_{43}b_{32})^2\sigma_{uu} + b_{43}^2\sigma_{vv} + \sigma_{ww}$$

Suppose we regard the expressions for the variances and covariances of the reduced-form disturbances, taking these quantities as known, as six equations in the six unknowns, σ_{uu}, σ_{vv}, σ_{ww}, b_{43}, b_{42}, and b_{32}. Although the equations involve nonlinear combinations of the unknowns, they are readily solved by a sequence of simple substitutions. Having solved for these parameters, we could return to the three equations defining the *a*'s (taking them as known) and solve for the remaining unknown structural coefficients, b_{41}, b_{31}, and b_{21}.

Thus, if the reduced-form coefficients (the *a*'s) and the variance–covariance matrix of the reduced-form disturbances were known, we could solve for structural coefficients (the *b*'s) and the variances of structural disturbances. From a computational point of view, this result is of no great practical value. Because of its conceptual interest, however, we indicate how one might proceed. Multiplying each reduced-form equation through by the exogenous variable and taking expectations, we obtain:

$$\sigma_{12} = a_{21}\sigma_{11}$$
$$\sigma_{13} = a_{31}\sigma_{11}$$
$$\sigma_{14} = a_{41}\sigma_{11}$$

Least squares estimates of the a's are, therefore,

$$\hat{a}_{21} = \frac{m_{12}}{m_{11}}$$

$$\hat{a}_{31} = \frac{m_{13}}{m_{11}}$$

$$\hat{a}_{41} = \frac{m_{14}}{m_{11}}$$

Multiplying through each reduced-form equation by each dependent variable and each reduced-form disturbance yields (after a little algebraic manipulation, which may serve as an exercise for the interested reader):

$$\sigma_{u'u'} = \sigma_{22} - a_{21}\sigma_{12}$$

$$\sigma_{v'v'} = \sigma_{33} - a_{31}\sigma_{13}$$

$$\sigma_{w'w'} = \sigma_{44} - a_{41}\sigma_{14}$$

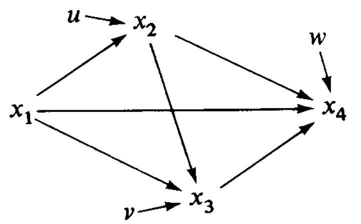
$$\sigma_{u'v'} = \sigma_{23} - a_{21}\sigma_{13}$$

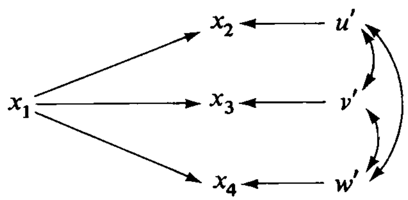
$$\sigma_{u'w'} = \sigma_{24} - a_{21}\sigma_{14}$$

$$\sigma_{v'w'} = \sigma_{34} - a_{31}\sigma_{14}$$

Thus, if we combine sample estimates of the variances and covariances of our observed variables with the least-squares estimates of the a's, we will generate estimates of the reduced-form disturbance variances and covariances. Putting these through the solution routine outlined earlier will yield estimates of structural coefficients and structural-form disturbance variances precisely the same as the estimates obtained by direct least-squares estimation of the structural equations themselves.

Comparing the path diagrams of the structural and reduced forms of the model may put some of these results in perspective:





The diagrams are equivalent in one sense for, as we have shown, given the parameters (coefficients, variances, and covariances of disturbances) of one form, we may solve for the parameters of the other. Each diagram depicts a model with ten parameters. In the structural form we have:

one variance of the exogenous variable
six structural coefficients
three variances of disturbances

In the reduced form we have:

one variance of the exogenous variable
three reduced-form coefficients
three variances of reduced-form disturbances
three covariances among reduced form disturbances

The paths in the reduced-form diagram represent (typically) some combination of compound paths in the structural-form diagram. This fact is an instructive implication of our definitions of the a's (see Figure 4.1).

Thus, the reduced-form coefficients sum up the several direct and indirect paths through which the exogenous variable exerts its effects on each dependent variable. If one cared to know only the *total* effect of the exogenous variable on a dependent variable, the reduced-form coefficient tells the whole story. But if one is interested in how that effect comes about, the greater detail of the structural model is informative. After all, in the reduced form, a great deal of the "structure" is buried in the rather uninformative variances and covariances of the reduced-form disturbances.

Exercise. *Derive the reduced form for a two-equation model consisting only of the x_3-equation and the x_4-equation in the model just discussed. Both x_1 and x_2 are exogenous, so that the variances of both and their covariance as well must be assumed to arise from exogenous sources. Express the variances and covariances of the reduced-form disturbances in terms of structural coefficients and structural-form disturbance variances. Show how the direct and indirect effects of exogenous variables (insofar as these are explicit in the model) are summed up in reduced-form coefficients. Compare the number of parameters in the structural and reduced forms.*

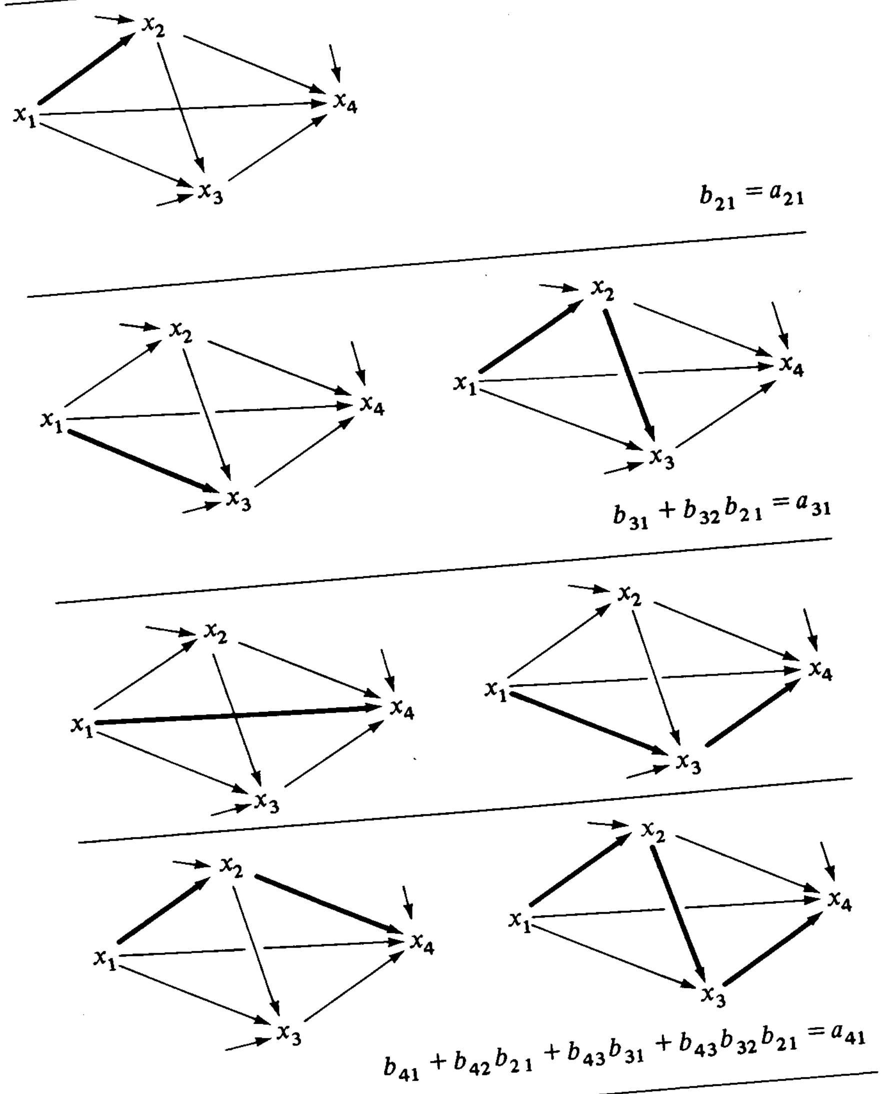


FIG. 4.1. How structural coefficients combine into reduced form coefficients in a fully recursive model

We have managed to postpone to this point a matter that many sociologists consider—erroneously, we believe—to be the single most important feature of a model, the proportion of variation in each dependent variable that is "explained." Although this emphasis upon the misnamed "explanatory power" of a model is mistaken, there is a limited utility in the multiple-correlation statistic. We proceed to indicate how it fits into the account of recursive models offered here.

Suppose the structural coefficients of the model (page 52) have been estimated by OLS. This method minimizes the sum of squares of the sample residuals and thereby insures that the residual from an estimated regression equation is uncorrelated with the regressors in that equation. Hence we have $\sum x_1 \hat{u} = \sum x_2 \hat{v} = \sum x_1 \hat{v} = \sum x_3 \hat{w} = \sum x_2 \hat{w} = \sum x_1 \hat{w} = 0$, where the summation is over the entire sample and $\hat{u}$, $\hat{v}$, and $\hat{w}$ are the sample residuals that estimate the corresponding disturbances. This allows us to operate on the equations of the model, when the structural coefficients therein are replaced by their OLS estimates, in much the same way that we have hitherto worked with the model itself. Thus, *in the sample*, we have

$$x_2 = \hat{b}_{21}x_1 + \hat{u}$$
$$x_3 = \hat{b}_{32}x_2 + \hat{b}_{31}x_1 + \hat{v}$$
$$x_4 = \hat{b}_{43}x_3 + \hat{b}_{42}x_2 + \hat{b}_{41}x_1 + \hat{w}$$

(These are, in effect, the formulas for computing $\hat{u}$, $\hat{v}$, and $\hat{w}$, respectively.) Each variable will have been expressed as a deviation from its sample mean. Multiplying each equation through by the residual and the dependent variable, we find

$$\sum x_2\hat{u} = m_{\hat{u}\hat{u}}$$
$$\sum x_3\hat{v} = m_{\hat{v}\hat{v}}$$
$$\sum x_4\hat{w} = m_{\hat{w}\hat{w}}$$
$$m_{22} = \hat{b}_{21}m_{12} + m_{\hat{u}\hat{u}}$$
$$m_{33} = \hat{b}_{32}m_{23} + \hat{b}_{31}m_{13} + m_{\hat{v}\hat{v}}$$
$$m_{44} = \hat{b}_{43}m_{34} + \hat{b}_{42}m_{24} + \hat{b}_{41}m_{14} + m_{\hat{w}\hat{w}}$$

The respective *coefficients of determination* for the three equations are

$$R^2_{2(1)} = 1 - \frac{m_{\hat{u}\hat{u}}}{m_{22}}$$

$$R^2_{3(21)} = 1 - \frac{m_{\hat{v}\hat{v}}}{m_{33}}$$

$$R^2_{4(321)} = 1 - \frac{m_{\hat{w}\hat{w}}}{m_{44}}$$

(The *multiple correlation* is the square root of R^2.) If we wished to put our results in the framework of standardized variables and path coefficients, we would proceed to note that

$$\hat{p}_{2u} = \sqrt{1 - R^2_{2(1)}}$$
$$\hat{p}_{3v} = \sqrt{1 - R^2_{3(21)}}$$
$$\hat{p}_{4w} = \sqrt{1 - R^2_{4(321)}}$$

which are the so-called residual paths.

It will be seen that the definition of R^2 rests on the distinction between variation "explained" by an equation in the model and the "unexplained" variation of that equation's dependent variable. It is sometimes suggested that the formula defining R^2 be used to effect a partitioning of the explained variation into portions due uniquely to the several determining causes. Thus, according to this suggestion, $R^2_{3(21)}$ (for example) would be allocated between x_1 and x_2 in proportion to $\hat{b}_{31} m_{13}$ and $\hat{b}_{32} m_{23}$, respectively (or, in the framework of path coefficients, $\hat{p}_{31} r_{13}$ and $\hat{p}_{32} r_{23}$). But the suggestion is mistaken. It is true that $\hat{b}_{31}$ estimates the direct effect of x_1 on x_3, but m_{13} reflects a mixture of effects arising from diverse sources. If we multiply through our estimate of the x_3-equation by x_1, we find

$$m_{13} = \hat{b}_{32} m_{12} + \hat{b}_{31} m_{11}$$

Hence

$$\hat{b}_{31} m_{13} = \hat{b}_{32} \hat{b}_{31} m_{12} + \hat{b}^2_{31} m_{11}$$

and the first term on the right is certainly not an unalloyed indicator of the role of x_1 alone in producing variation in x_3.

Indeed the "problem" of partitioning R^2 bears no essential relationship to estimating or testing a model, and it really does not add anything to our understanding of how a model works. The simplest recommendation—one which saves both work and worry—is to eschew altogether the task of dividing up R^2 into unique causal components. In a strict sense, it just cannot be done, even though many sociologists, psychologists, and other quixotic persons cannot be persuaded to forego the attempt.

Indeed the whole issue of what to make of a multiple correlation is clarified by noting that R^2 does not estimate any parameter of the model as such, but rather a parameter that depends upon both the model and the population in which the model applies. We have no reason to expect R^2 to be invariant across populations. If either σ_{ww} or σ_{11} (and, therefore, σ_{44}) changes in going to a new population, while the structural coefficients remain fixed, R^2 will be changed. This is a matter of special concern in the event that the value of σ_{11} is essentially under the investigator's control, according to whether he (for example) puts greater or lesser variance into the distribution of his experimental stimulus x_1, or samples disproportionately from different parts of the "natural" range of x_1. As we have seen, such a modification of σ_{11} will change all variances and covariances, as well as R^2, even though there is no change in structural coefficients—and there is no reason to expect them to be affected by either of these aspects of study design.

The real utility of R^2 is that it tells us something about the precision of our estimates of coefficients, since the standard error of a coefficient is a function, among other things, of R^2.

It is a mistake—the kind of mistake easily made by the novice—to focus too much attention on the magnitude of R^2. *Other things being equal*, it is, of course, true that one prefers a model yielding a high R^2 to one yielding a lower value. But the *ceteris paribus* clause is terribly important. Merely increasing R^2 by lengthening the list of regressors is no great achievement unless the role of those variables in an extended causal model is properly understood and correctly represented.

Suppose, for example, that in using the four-variable recursive model we have been studying, the investigator became dissatisfied with the low value of $R^2_{3(21)}$. It would be quite easy to get a higher value of R^2, for example, by running the regression of x_3 on x_4, x_2, and x_1, and reporting the value of $R^2_{3(421)}$. But this regression does not correspond to the causal ordering of the variables, which was required to be specified at the outset. It is, therefore, an exceedingly misleading statistic. (One does not often see the mistake in quite this crude a form. But naïvely regressing causes on effects is far from being unknown in the literature.)

Another way to raise $R^2_{3(21)}$ would be to introduce another variable, say x'_3, that is essentially an alternative measure of x_3, though giving slightly different results. The regression of x_3 on x'_3, x_2, and x_1 is then guaranteed to yield a high value of R^2.

Indeed, the best-known examples of very high correlations are those selected to convey the notion of "spurious correlation," "nonsense correlation in time series," or other kinds of artifact. This shows us that high values of R^2, in themselves, are not sufficient to evaluate a model as successful.

Before worrying too much about his R^2, therefore, the investigator does well to reconsider the entire specification of the model. If that specification cannot be faulted on other grounds, the R^2 as such is not sufficient reason to call it into question.

Exercise. *To conclude, for the time being, your study of fully recursive models, review the material on estimation and testing in Chapter 3 and restate the essential points so that they apply to the recursive model as expressed without standardization of variables (page 52).*

FURTHER READING

An example of a recursive sociological model presented in terms of both standardized and nonstandardized coefficients appears in Duncan (1969). Note that the more interesting conclusions were developed on the basis of the latter. On the questionable value of commonly used measures of "relative importance" or "unique contribution" of the several variables in an equation, see Ward (1969), Cain and Watts (1970), and Duncan (1970).

5

A Just-Identified Nonrecursive Model

The model considered throughout this chapter is

$$x_3 = b_{31}x_1 + b_{34}x_4 + u$$

$$x_4 = b_{42}x_2 + b_{43}x_3 + v$$

For convenience, $E(x_j) = 0$, $j = 1, \ldots, 4$, and $E(u) = E(v) = 0$. However, we do *not* put the variables in standard form. Variables x_1 and x_2 are *exogenous*; their variances and their covariance are not explained within the model. Variables x_3 and x_4 are *jointly dependent* or *endogenous*; the purpose of the model is to explain the behavior of these variables. Variables u and v are, respectively, the *disturbances* in the x_3-equation and the x_4-equation. Their presence accounts for the fact that x_3 and x_4 are not fully explained by their explicit determining factors. The model will be operational only if we can assume that disturbances are uncorrelated with exogenous variables; hence the specification $E(x_1 u) = E(x_1 v) = E(x_2 u) = E(x_2 v) = 0$. *This is a serious assumption.* The research worker must carefully consider what circumstances would violate it and whether his theoretical understanding of the situation under study permits him to rule out such violations.

In contrast to the case of a fully recursive model, in the nonrecursive model the specification of zero covariances between disturbances and