

STA402S98 Formula Sheet for Test 2

$$E[g(Y) | X=x] = \sum_y g(y) f_{Y|X}(y|x) = \sum_y g(y) \frac{P(Y=y \cap X=x)}{P(X=x)} \quad (\text{Treat these as definitions})$$

$$E[g(Y)] = E[E[g(Y) | X=x]] = \sum_x E[g(Y) | X=x] f_X(x)$$

$$E[Y | A=i] = \sum_{j=1}^b E[Y | A=i \cap B=j] P(B=j | A=i) = \sum_{j=1}^b \mu_{i,j} P(B=j | A=i)$$

For the multiple linear regression model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, $\mathbf{X}$ ($n \times p$) known constant, $\boldsymbol{\beta}$ ($p \times 1$) unknown constant, $\boldsymbol{\varepsilon}$ ($n \times 1$) error random vector, we are often interested in testing $H_0: \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$, where $\mathbf{C}$ is an $s \times p$ matrix of rank $s \leq p$.

$$\text{For a single-factor design, } L = \sum_{i=1}^r c_i \mu_i; \quad \hat{L} = \sum_{i=1}^r c_i \bar{Y}_{i\cdot}; \quad s\{\hat{L}\} = \sqrt{\text{MSE} \sum_{i=1}^r \frac{c_i^2}{n_i}}$$

$$\text{Bonferroni: } \hat{L} \pm B \quad s\{\hat{L}\}, \quad B = t(1 - \frac{\alpha}{2g}; n_T - r)$$

$$\text{Scheffé: } \hat{L} \pm S \quad s\{\hat{L}\}, \quad S = \sqrt{(r-1) F(1 - \alpha; r-1, n_T - r)}$$

For two-factor ANOVA model with factors A (a levels) and B (b levels),

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} = \mu_{ij} + \varepsilon_{ijk}, \quad \text{where } \varepsilon_{ijk} \text{ are independent } N(0, \sigma^2)$$

$$\mu_{\cdot j} = \frac{\sum_{i=1}^a \mu_{ij}}{a}, \quad \mu_{i\cdot} = \frac{\sum_{j=1}^b \mu_{ij}}{b}, \quad \mu_{..} = \frac{\sum_{i=1}^a \mu_{i\cdot}}{a} = \frac{\sum_{j=1}^b \mu_{\cdot j}}{b}$$

$$\alpha_i = \mu_{i\cdot} - \mu_{..}, \quad \beta_j = \mu_{\cdot j} - \mu_{..}, \quad (\alpha\beta)_{ij} = \mu_{ij} - (\mu_{i\cdot} + \mu_{\cdot j} - \mu_{..})$$

When all cell sample sizes = n,

$$Y_{ij\cdot} = \sum_{k=1}^n Y_{ijk}; \quad Y_{i\cdot\cdot} = \sum_{j=1}^b \sum_{k=1}^n Y_{ijk}; \quad Y_{\dots} = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n Y_{ijk}$$

$$\bar{Y}_{ij\cdot} = Y_{ij\cdot}/n; \quad \bar{Y}_{i\cdot\cdot} = Y_{i\cdot\cdot}/(bn); \quad \bar{Y}_{\cdot j\cdot} = Y_{\cdot j\cdot}/(an); \quad \bar{Y}_{\dots} = Y_{\dots}/(abn)$$