STA 378 Assignment 1

1. Let the test statistic T be continuous, with pdf $f(t)$ and cdf $F(t)$ under the null hypothesis. The null hypothesis is rejected if $T > c$. Show that if H_0 is true, the distribution of the p-value is $U(0,1)$. Derive the density. Start with the cumulative distribution function of the p-value: $Pr\{P \leq x\} = \ldots$.

2. Suppose H_0 is false. Would you expect the distribution of the p-value to still be uniform? Pick one of the alternatives below. You are not asked to derive anything for now.

   (a) The distribution should still be uniform.

   (b) We would expect more small p-values.

   (c) We would expect more large p-values.

3. Let $P_i \sim U(0,1)$. Show that $Y_i = -2\ln(P_i)$ has a χ^2 distribution. What are the degrees of freedom?

4. Let $P_1, \ldots P_n$ be a random sample of p-values with the null hypotheses all true, and let $Y = \sum_{i=1}^{n} -2\ln(P_i)$. What is the distribution of Y? Only derive it (using moment-generating functions) if you don't know the answer.

5. Let $P_i \sim U(0,1)$, and denote the cumulative distribution function of the standard normal by $\Phi(x)$.

   (a) What is the distribution of $Y_i = \Phi^{-1}(1 - P_i)$? Show your work.

   (b) If H_0 is false and P_i is not uniform, would you expect Y_i to be bigger, or smaller? Why?

6. Let $P_1, \ldots P_n$ be a random sample of p-values.

   (a) Propose a test statistic based on your answer to Question 5a.

   (b) What is the null hypothesis of your test?

   (c) What is the distribution of your test statistic under the null hypothesis? Only derive it (using moment-generating functions) if you don't know the answer.

   (d) Would you reject the null hypotesis when your test statistic has big values, or when it has small values? Which one?

7. Suppose we observe the following random sample of p-values: `0.016 0.188 0.638 0.148 0.917 0.124 0.695`.

   (a) For the test statistic of Question 4,

      i. What is the critical value at $\alpha = 0.05$? The answer is a number.

      ii. What is the value of the test statistic? The answer is a number.

      iii. Do you reject the null hypothesis? Yes or No.

      iv. What if anything do you conclude?

   (b) For the test statistic of Question 5a,

      i. What is the critical value at $\alpha = 0.05$? The answer is a number.

      ii. What is the value of the test statistic? The answer is a number.

      iii. Do you reject the null hypothesis? Yes or No.

      iv. What if anything do you conclude?