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Student Number

STA 312 s2019 Quiz 1

The Exponential(λ) distribution has density $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$, where $\lambda > 0$. Let the random variable X have an exponential distribution.

1. (4 points) Find $P(X > t)$ for $t > 0$. Show your work, including the change of variables. **Circle your answer.**

$$P(X > t) = \int_0^t \lambda e^{-\lambda x}\, dx$$

$u = -\lambda x$

$du = -\lambda\, dx$

x	u
t	$-\lambda t$
0	0

$$= -\int_0^{-\lambda t} e^u\, du$$

$$= -e^u \Big|_0^{-\lambda t} = (-1)(e^{-\lambda t} - 1)$$

$$= 1 - e^{-\lambda t}$$

They don't have to do the change of variables exactly this way.

2. (6 points) Prove the "memoryless" property of the exponential distribution. For $t > 0$ and $s > 0$, show

$$P(X > t + s | X > s) = P(X > t)$$

Warning: If you "force" the answer to come out right by by making a deliberate mistake, you will get zero marks on this question.

$$P(X > t+s \,/\, X > t) = \frac{P(X > t+s,\ X > s)}{P(X > s)}$$

$$= \frac{P(X > t+s)}{P(X > s)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}}$$

$$= \frac{e^{-\lambda t - \lambda s}}{e^{-\lambda s}} = \frac{e^{-\lambda t}\cancel{e^{-\lambda s}}}{\cancel{e^{-\lambda s}}}$$

$$= e^{-\lambda t} = P(X > t)$$

They do not need to show this much detail.