Name Jerry

Student Number

STA 312f2012 Quiz 3

1. (5 points) Recall the coffee taste test study, in which 60 of 100 consumers preferred the new blend of coffee beans. Please test the null hypothesis of equal preference using a likelihood ratio test.

   - Calculate the test statistic. Show your work. The final answer is a number. **Circle the number.**

Under H_0, expected frequencies are $\hat{\underset{\sim}{\mu}} = n(\pi_0, 1-\pi_0)$

$= 100(\frac{1}{2}, \frac{1}{2}) = (50, 50)$. So

$G^2 = 2\left(60 \log\left(\frac{60}{50}\right) + 40 \log\left(\frac{40}{50}\right)\right) = 4.03$

   - What is the critical value of the test statistic? The answer is a number. 3.841

   - Do you reject H_0 at $\alpha = 0.05$? Answer Yes or No. Yes

2. (5 points) The last question of your homework was a power calculation for the Big Red gum study. You were asked for the sample size that would yield a power of 0.90 when true brand awareness was 0.08. What is the required sample size for the Pearson X^2 test? The answer is a number from your printout. **Write the number on this paper in the space below.**

n = 721

Also, circle the number — *only one number* on your printout. No marks without the correct number circled on the printout.

Attach the printout to this paper. Make sure your name is on the printout.

Formulas

$Z_1 = \frac{\sqrt{n}(p-\pi_0)}{\sqrt{\pi_0(1-\pi_0)}}$ $\quad Z_2 = \frac{\sqrt{n}(p-\pi_0)}{\sqrt{p(1-p)}}$ $\quad p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$

```
> qnorm(0.975)
[1] 1.959964
> qnorm(0.995)
[1] 2.575829
```

$P(n_1, \ldots, n_c) = \binom{n}{n_1 \cdots n_c} \pi_1^{n_1} \cdots \pi_c^{n_c}$

$\ell(\boldsymbol{\pi}) = \prod_{i=1}^{n} \pi_1^{y_{i,1}} \pi_2^{y_{i,2}} \cdots \pi_c^{y_{i,c}} = \pi_1^{n_1} \pi_2^{n_2} \cdots \pi_c^{n_c}$

$G^2 = -2\log\left(\frac{\max_{\beta \in \mathcal{B}_0} \ell(\beta)}{\max_{\beta \in \mathcal{B}} \ell(\beta)}\right) = -2\log\left(\frac{\ell_0}{\ell_1}\right)$

$G^2 = 2\sum_{j=1}^{c} n_j \log\left(\frac{n_j}{n\widehat{\pi}_j}\right) = 2\sum_{j=1}^{c} n_j \log\left(\frac{n_j}{\widehat{\mu}_j}\right)$

$X^2 = \sum_{j=1}^{c} \frac{(n_j - \widehat{\mu}_j)^2}{\widehat{\mu}_j}$

$$\lambda = n\sum_{j=1}^{c} \frac{[\pi_j - \pi_j(M)]^2}{\pi_j(M)}$$

$$\lambda = 2n\sum_{j=1}^{c} \pi_j \log\left(\frac{\pi_j}{\pi_j(M)}\right)$$

```
> df = 1:8
> CriticalValue = qchisq(0.95,df)
> round(rbind(df,CriticalValue),3)
               [,1]  [,2]  [,3]  [,4]  [,5]   [,6]   [,7]   [,8]
df            1.000 2.000 3.000 4.000  5.00  6.000  7.000  8.000
CriticalValue 3.841 5.991 7.815 9.488 11.07 12.592 14.067 15.507
```

```
R version 2.13.2 (2011-09-30)
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Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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> # R work for Question 2
>
> piM = c(0.06,0.94) # Exact null hypothesis
> pi = c(0.08,0.92)  # Truth (They hope)
> critval = qchisq(0.95,1)
>
> n = 0; power = 0
> while(power < 0.90)
+     {   n = n+1
+         lambda = n * sum( (pi-piM)^2/piM )
+         power = power = 1-pchisq(critval,5,lambda)
+     }
> n; power
[1] 721
[1] 0.9001803
>
>
```