STA 312f10 Final Exam Formulas

$L(\mathbf{p}) = p_1^{x_1} \cdots p_k^{x_k} = p_1^{x_1} \cdots p_{k-1}^{x_{k-1}} (1 - \sum_{j=1}^{k-1} p_j)^{N - \sum_{j=1}^{k-1} x_j}$

$G^2 = -2 \log \left(\frac{\max_{\boldsymbol{\theta} \in \Theta_0} L(\boldsymbol{\theta})}{\max_{\boldsymbol{\theta} \in \Theta} L(\boldsymbol{\theta})} \right) = -2 \log \left(\frac{L(\widehat{\boldsymbol{\theta}}_0)}{L(\widehat{\boldsymbol{\theta}})} \right) \quad \widehat{\mathbf{m}} = N \, \widehat{\boldsymbol{\theta}}_0$

$G^2 = 2 \sum (\text{Observed}) \log \left(\frac{\text{Observed}}{\text{Expected}} \right) \qquad X^2 = \sum \frac{(\text{Observed-Expected})^2}{\text{Expected}}$

$\log m_{ij} = \mu + \mu_{1(i)} + \mu_{2(j)} + \mu_{12(ij)} \qquad \mu = \frac{1}{IJ} \sum_{i=1}^{I} \sum_{j=1}^{J} \log m_{ij}$

$\mu_{1(i)} = \frac{1}{J} \sum_{j=1}^{J} \log m_{ij} - \mu \qquad \mu_{2(j)} = \frac{1}{I} \sum_{i=1}^{I} \log m_{ij} - \mu$

$\alpha = \frac{m_{11} m_{22}}{m_{12} m_{21}} = \frac{p_{11} p_{22}}{p_{12} p_{21}} = e^{4 \mu_{12(11)}} \qquad \mu_{12(11)} = \frac{\log \alpha}{4}$

$$\begin{aligned} \log m_{ijk} = \mu \quad &+ \quad \mu_{1(i)} + \mu_{2(j)} + \mu_{3(k)} \\ &+ \quad \mu_{12(ij)} + \mu_{13(ik)} + \mu_{23(jk)} \\ &+ \quad \mu_{123(ijk)} \end{aligned}$$

$\log m = \mu \ + \ \mu_1 + \mu_2 + \mu_3 \ + \ \mu_{12} + \mu_{13} + \mu_{23} \ + \ \mu_{123}.$

$\log \frac{\pi}{1-\pi} = \beta_0 + \beta_0 x_1 + \ldots + \beta_{p-1} x_{p-1} = \mathbf{x}' \boldsymbol{\beta} \Leftrightarrow \pi = \frac{e^{\mathbf{x}' \boldsymbol{\beta}}}{1 + e^{\mathbf{x}' \boldsymbol{\beta}}}$

$\log \lambda = \beta_0 + \beta_0 x_1 + \ldots + \beta_{p-1} x_{p-1}$

$\log(\pi_j / \pi_k) = \beta_{0,j} + \beta_{1,j} x_1 + \ldots + \beta_{p-1,j} x_{p-1}$ for $j = 1, \ldots, k-1$

$\boldsymbol{\mathcal{J}}(\boldsymbol{\theta})_{k \times k} = \left[-\frac{\partial^2}{\partial \theta_i \partial \theta_j} \ell(\boldsymbol{\theta}, \mathbf{Y}) \right] \qquad \widehat{\mathbf{V}} = \boldsymbol{\mathcal{J}}(\widehat{\boldsymbol{\theta}})^{-1}$

$W = (\mathbf{C} \widehat{\boldsymbol{\theta}} - \mathbf{h})' (\mathbf{C} \widehat{\mathbf{V}} \mathbf{C}')^{-1} (\mathbf{C} \widehat{\boldsymbol{\theta}} - \mathbf{h}) \quad S = \mathbf{u}(\widehat{\boldsymbol{\theta}}_0)' \boldsymbol{\mathcal{J}}(\widehat{\boldsymbol{\theta}}_0)^{-1} \mathbf{u}(\widehat{\boldsymbol{\theta}}_0), \ \mathbf{u}(\boldsymbol{\theta})_{k \times 1} = \left[\frac{\partial \ell}{\partial \theta_i} \right]$

```
> df = 1:8
> CriticalValue = qchisq(0.95,df)
> round(rbind(df,CriticalValue),3)
                [,1]  [,2]  [,3]  [,4]  [,5]   [,6]   [,7]   [,8]
df             1.000 2.000 3.000 4.000  5.00  6.000  7.000  8.000
CriticalValue 3.841 5.991 7.815 9.488 11.07 12.592 14.067 15.507
```