Name ________________________

Student Number ________________

# Test 1 STA302F94

Aids Allowed: Calculator; tables and formula sheet supplied

1. (2 pts) For a t-distribution with 20 degrees of freedom, what is the point x such that 95% of the distribution is below x?

2. (4 pts) An extremely light metal has been invented for use by the automobile industry. Tensile strength measurements are made on 15 pieces of the metal; their mean and sample standard deviation are 39.3 and 2.6, respectively. Assuming a normal distribution for tensile strength of this metal, find a 90% confidence interval for $\mu$.

3. (2 pts) Answer the following 3 questions "yes" or "no." You must get all three correct to get any credit.

   a. Suppose $\alpha$=.01 and p=.0013. Do you reject $H_0$?

   b. In general, if $p<\alpha$, do you reject $H_0$?

   c. If results are "statistically significant," do you reject $H_0$?

4. A test of perceptual accuracy is given to 61 individuals before and after a special training procedure designed to improve perceptual accuracy. The score on the test is number of mistakes, so the higher the score, the lower the accuracy. A scientist does a two-sided t–test (even though an argument could be made in this case for a one-sided test), using $\alpha$=.05.

a. (2 pts) What is the null hypothesis $H_0$? Give the answer in symbols, not words.

b. (2 pts) What is the alternative hypothesis $H_a$?

c. (2 pts) What is the decision rule?

d. (2 pts) Suppose we observe t* = 1.97. Do you reject $H_0$? Is the mean difference $\bar{Y}$ significantly different from zero at $\alpha$=.05?

e. (3 pts) If t*=1.97, what do you conclude about the effectiveness of the training procedure?

f. (2 pts) For this study, the maximum probability of rejecting $H_0$ just by chance if $H_0$ is true is ...? (Give a NUMBER).

5. (3 pts) Suppose that you believe that a special educational program will have some effect on the dropout rate of disadvantaged youngsters. State your null hypothesis in words, not symbols.

6. Here are some before-after results. You may assume a normal distribution.

| Before | After |
|---|---|
| 6 | 2 |
| 2 | 1 |
| 3 | 5 |
| 2 | 1 |

a. (4 pts) What is the value of t*?

b. (2 pts) What is the decision rule for a two–sided test at $\alpha$=0.05?

c. (1 pt) Do you reject $H_0$? at $\alpha$=0.05

d. (3 pts) What do you conclude about the difference between before and after scores.

7. Thirty-two white rats are assigned at random to either an experimental group or a control group. The experimental group is injected with anabolic steroids and the control group is given a sham injection of a saline solution. Each rat is placed in a cage with a hungry boa constrictor, and its survival time is recorded.

a. (2 pts) What is the independent variable?

b. (2 pts) What is the dependent variable?

c. (2 pts) For a two-sided t-test, $H_0$ would be rejected at $\alpha$ = .01
if |t*| > _______.

8. (5 pts) Let $\mathbf{X} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$. What is $\mathbf{XX'}$? (NOT $\mathbf{X'X}$!)

9. (6 pts) Let $\mathbf{X}$ be ANY nxp matrix. Prove that $\mathbf{X'X}$ is symmetric. If you give an example rather than a proof, you will get zero points.

10. Consider the following functions of the random variables $Y_1$, $Y_2$ and $Y_3$.

$$W_1 = Y_1$$
$$W_2 = Y_1 + Y_2$$
$$W_3 = Y_1 + Y_2 + Y_3$$

a. (5 pts) In matrix notation, $\mathbf{W}=\mathbf{AY}$. What is $\mathbf{A}$?

b. (2 pts) Denoting the expected value of the $\mathbf{Y}$ vector by $\boldsymbol{\mu}$, what is $E\{\mathbf{W}\}$? Use matrix notation

11. For simple linear regression, the **X** matrix may be written as $\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$.

a. (6 pts) Using summation notation where it is useful, write the $\mathbf{X'X}$ matrix in as simple a form as possible. (I want a square matrix, with its contents written in symbols).

b. (10 pts) Suppose that $x_1 = x_2 = ... = x_n = k$ (k is some real constant). Prove that $\mathbf{X'X}$ is singular.

12. (6 pts) Show by an example that if **A** and **B** are matrices of the right size (they have to be square), in general **AB≠BA**.

13. (2 pts) Suppose the standard deviation of X is half the standard deviation of Y, and r=.75. What is the slope of the least-squares regression line?

14. (2 pts) Suppose that the correlation between X & Y equals zero. Can you conclude that X and Y are unrelated? Answer in one sentence or phrase, but more than one word.

15. (2 pts) If r = –.70, there is a moderate to strong tendency for low values of X to go with ______ values of Y, and high values of X to go with ______ values of Y.

16. (2 pts) In simple linear regression, what is the relationship between the sign (positive or negative) of the correlation, and the sign of $b_1$?

17. (8 pts) Partially differentiate the sum of squares $Q(b_0,b_1) = \sum_{i=1}^{n}(Y_i - b_0 - b_1 x_i)^2$ with respect to $b_0$ and $b_1$, set the derivatives to zero, and obtain the normal equations.

18. (4 pts) In your homework, you wrote the normal equations as $\mathbf{X'Xb} = \mathbf{X'Y}$. Assuming $[\mathbf{X'X}]^{-1}$ exists, use matrix methods to solve for $\mathbf{b}$. Express your answer in matrix notation in terms of $\mathbf{X}$ and $\mathbf{Y}$ as defined previously. Yes, this is a very easy question. The answer is a one or two–liner. Do not make it harder than it is.

Total = 100 Points