We study the problem of nonparametric dependence detection when no assumption is made about the joint distribution. We approach this problem by introducing the new concept of binary expansion statistics (BEStat), which examines dependence through a filtration induced by marginal binary expansions. In particular, we propose the binary expansion testing (BET) framework to test independence up to certain depth in the binary expansions.

The BET connects the nonparametric dependence detection problem with four fundamental concepts in statistics---copula, filtration, orthogonal design and multiple testing. By combining the strength of these classical statistical wisdoms, the BET improves upon a wide class of commonly used methods (a) by avoiding the problems of the clustering intuition and non-uniform consistency and (b) by providing clear interpretations of global and local relationships upon rejection of independence.

The binary expansion approach also connects the test statistics with the current computing system to allow efficient bitwise implementations. We illustrate the BET by a study of the distribution of stars in the night sky and by an exploratory data analysis of the TCGA breast cancer data.