EXAMPLE 2.2

(*Accumulated value of an annuity at the time of final payment*)
What level amount must be deposited on May 1 and November 1 each year from 1998 to 2005, inclusive, to accumulate to 7000 on November 1, 2005 if the nominal annual rate of interest compounded semi-annually is 9%?

SOLUTION

A first step in translating the verbal description of this annuity into an algebraic form is to determine the number of deposits being made. There are a total of 16 deposits (2 per year for each of the 8 years from 1998 to 2005 inclusive) and they occur every $\frac{1}{2}$-year. As a second step, we note that the $\frac{1}{2}$-year interest rate is 4.5%, and the $\frac{1}{2}$-year payment period corresponds to the $\frac{1}{2}$-year interest compounding period. If the level amount deposited every $\frac{1}{2}$-year is denoted by X, the accumulated value of the deposits at the time of the 16^{th} deposit is

$$X \cdot \left[(1.045)^{15} + (1.045)^{14} + \cdots + 1.045 + 1\right]$$

$$= X \cdot \frac{(1.045)^{16} - 1}{.045} = X \cdot s_{\overline{16}|.045} = 22.719337X$$

(note that the factor $(1.045)^{15}$ arises as a result of there being 15 half-year periods from the time of the first deposit on May 1, 1998 to the time of the 16^{th} deposit on November 1, 2005). Then

$$X = \frac{7000}{s_{\overline{16}|.045}} = \frac{7000}{22.719337} = 308.11. \qquad \square$$

All financial calculators have functions that calculate the accumulated value of an annuity at the time of the final payment if the payment amount, number of payments, and interest rate are known. The calculator comments in the Appendix provide a number of examples of this for two popular financial calculators, the Texas Instruments BA II PLUS and the Hewlett-Packard HP-12C.