Name Jerry

Student Number

STA 431s13 Quiz 5

In the following model, X_i, $Y_{i,1}$ and $Y_{i,2}$ are latent variables, while only W_i, $V_{i,1}$ and $V_{i,2}$ are observable. Independently for $i = 1, \ldots, n$, let

$$\begin{aligned} Y_{i,1} &= \beta_1 X_i + \epsilon_{i,1} \\ Y_{i,2} &= \beta_2 X_i + \epsilon_{i,2} \\ V_{i,1} &= Y_{i,1} + e_{i,1} \\ V_{i,2} &= Y_{i,2} + e_{i,2} \\ W_i &= X_i + e_{i,3}, \end{aligned}$$

where $X_i \sim N(0, \phi)$, $\epsilon_{i,1} \sim N(0, \psi_1)$, $\epsilon_{i,2} \sim N(0, \psi_2)$, $e_{i,1} \sim N(0, \omega_1)$, $e_{i,2} \sim N(0, \omega_2)$, $e_{i,3} \sim N(0, \omega_3)$, and all these variables are independent.

1. (1 point) What is the parameter vector $\boldsymbol{\theta}$ for this model?

$\underset{\sim}{\theta} = (\beta_1, \beta_2, \phi, \psi_1, \psi_2, \omega_1, \omega_2, \omega_3)$

2. (1 point) Does this problem pass the test of the Parameter Count Rule? Answer Yes or No and give the numbers.

No. There are 8 parameters, but only 6 covariance structure equations.

3. (3 points) Calculate $Cov(V_{i,1}, V_{i,2})$. Show your work. Note that the variables are already centered.

$Cov(V_1, V_2) = E(V_1 V_2) = E(Y_1 + e_1)(Y_2 + e_2)$

$= E(\beta_1 X + \varepsilon_1 + e_1)(\beta_2 X + \varepsilon_2 + e_2)$

$= \beta_1 \beta_2 E(X^2) + 0$

$= \beta_1 \beta_2 \phi$

$W = X + e_3$

$V_1 = \beta_1 X + \varepsilon_1 + e_1$

$V_2 = \beta_2 X + \varepsilon_2 + e_2$

4. (2 points) Write down the covariance matrix of the observable variables, which of course includes your answer to the preceding question. You *do not* need to show your work for this part.

	W	V_1	V_2
W	$\phi + \omega_3$	$\beta_1 \phi$	$\beta_2 \phi = \sigma_{13}$
V_1		$\beta_1^2 \phi + \psi_1 + \omega_1$	$\beta_1 \beta_2 \phi = \sigma_{23}$
V_2			$\beta_2^2 \phi + \psi_2 + \omega_2$

5. (3 points) Assume that $Y_{i,2}$ is a well-chosen instrumental variable, so that $\beta_2 \neq 0$. Under this assumption, is β_1 identifiable? **Answer Yes or No and circle your answer.** Prove it. If the answer is Yes, show how β_1 can be recovered from the covariance matrix. If the answer is no, give a numerical of two parameter vectors with $\beta_2 \neq 0$, different values of β_1, and exactly the same covariance matrix $[\sigma_{ij}]$.

Yes : $\beta_1 = \dfrac{\sigma_{23}}{\sigma_{13}}$