STA302F94: Formula Sheet for Final Exam

$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ $\quad \hat{\mathbf{Y}} = \mathbf{Xb} = \mathbf{HY}$, $\quad$ where $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$

$\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}}$, $\quad$ SSE $= \mathbf{e}'\mathbf{e}$, $\quad$ MSE = SSE/(n–p), $\quad$ E(MSE) $= \sigma^2$

$$SSTO = \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \qquad SSE = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 , \qquad SSR = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2$$

SSTO = SSR + SSE, $\quad R^2 = \frac{SSR}{SSTO}$, dfto = n–1, dfr = p–1, dfe = n–p

MVN1: If $\mathbf{W}$ is a kx1 MVN($\boldsymbol{\mu},\boldsymbol{\Sigma}$) random vector and $\mathbf{A}$ is a kx1 matrix of constants, then $\mathbf{W} + \mathbf{A} \sim$ MVN($\mathbf{A}+\boldsymbol{\mu},\boldsymbol{\Sigma}$)

MVN2: If $\mathbf{W}$ is a kx1 MVN($\boldsymbol{\mu},\boldsymbol{\Sigma}$) random vector and $\mathbf{A}$ is an rxk matrix of constants with row rank $r \leq k$, then $\mathbf{AW}$ is MVN($\mathbf{A}\boldsymbol{\mu},\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'$).

$$\mathbf{s}^2\{\mathbf{b}\} = MSE(\mathbf{X}'\mathbf{X})^{-1} \qquad \frac{b_k - \beta_k}{s\{b_k\}} \sim t(n\text{-}p), \quad t^* = \frac{b_k}{s\{b_k\}} , \quad b_k \pm t(1\text{-}\alpha/2;n\text{-}p)\, s\{b_k\}$$

F^* = MSR/MSE ~ F(p-1,n-p) under H_0: $\beta_1 = \ldots = \beta_{p-1} = 0$

$F^* = \frac{SSE(R) - SSE(F)}{dfe_R - dfe_F} \Big/ MSE(F) \sim F(dfe_F\text{-}dfe_R, dfe_F)$ under H_0 that the reduced model is true.

Useful reduced model: $\mathbf{C}\boldsymbol{\beta} = \mathbf{h}$, where $\mathbf{C}$ is an sxp matrix of rank s. In this case $dfe_F\text{-}dfe_R = s$.

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proc reg;
     model mpg = c1 c2 weight length; /* Full model */
     model mpg = weight length; /* Reduced model */
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$$\mathbf{x}_h = \begin{bmatrix} 1 \\ x_{h,1} \\ \vdots \\ x_{h,p-1} \end{bmatrix}, \ E(Y_h) = \mathbf{x}_h' \boldsymbol{\beta}, \ \hat{Y}_h = \mathbf{x}_h' \mathbf{b}$$

C.I. for $E(Y_h)$: $\hat{Y}_h \pm t(1-\alpha/2;n-p) \sqrt{MSE\, \mathbf{x}_h'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_h}$

P.I. for $Y_{h(new)}$: $\hat{Y}_h \pm t(1-\alpha/2;n-p) \sqrt{MSE\,(1 + \mathbf{x}_h'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_h)}$

Bonferroni correction for k tests or C.I.s: Replace α by α/k

Leverage is considered big if diagonal element of $\mathbf{H}$, $h_{i,i} > n/p$

$e_i^* = \dfrac{e_i}{\sqrt{MSE(1 - h_{i,i})}}$ "Internally" studentized residual

$d_i = Y_i - \hat{Y}_{i\,(i)} = \dfrac{e_i}{(1 - h_{i,i})}$, $s^2\{d_i\} = MSE_{(i)}(1 + \mathbf{X}_{(i)}(\mathbf{X}'_{(i)}\mathbf{X}_{(i)})^{-1}\mathbf{X}'_{(i)})$

$\dfrac{d_i}{s\{d_i\}} \sim t(n-p-1)$ Studentized deleted residual

$DFFITS = \dfrac{\hat{Y}_i - \hat{Y}_{i\,(i)}}{\sqrt{MSE_{(i)}\, h_{i,i}}}$ is considered big if > 1 for small to moderate n, > $2\sqrt{p/n}$ for large n

$(DFBETAS)_{k(i)} = \dfrac{b_k - b_{k\,(i)}}{\sqrt{MSE_{(i)}\, c_{k,k}}}$, where $c_{k,k}$ is kth diagonal element of $(\mathbf{X}'\mathbf{X})^{-1}$ is considered big if > 1 for small to moderate n, > $2/\sqrt{n}$ for large n

$D_i = \dfrac{(\mathbf{b} - \mathbf{b}_{(i)})'(\mathbf{X}'\mathbf{X})(\mathbf{b} - \mathbf{b}_{(i)})}{p\,MSE}$ Cook's distance is considered big if > 50th %ile of F(p,n–p)

If $\sigma^2\{\varepsilon_i\} = \sigma^2\{Y_i\} = \sigma^2/w_i$, minimize $Q_w(\mathbf{b}) = \sum_{i=1}^{n} w_i\,(Y_i - \hat{Y}_i)^2$ by solving normal equations

$\mathbf{X}'\mathbf{W}\mathbf{X}\mathbf{b} = \mathbf{X}'\mathbf{W}\mathbf{Y} \Leftrightarrow \mathbf{b} = (\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{Y}$

Tolerance of an IV= $1-R^2$, where R^2 is the proportion of variation <u>in the IV</u> explained by the other IVs in the model.